

Shear viscosity in magnetized neutron star crust

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Abstract – The electron shear viscosity due to Coulomb scattering of degenerate electrons by atomic nuclei throughout a magnetized neutron star crust is calculated. The theory is based on the shear viscosity coefficient calculated neglecting magnetic fields but taking into account gaseous, liquid and solid states of atomic nuclei, multiphonon scattering processes, and finite sizes of the nuclei albeit neglecting the effects of electron band structure. The effects of strong magnetic fields are included in the relaxation time approximation with the effective electron relaxation time taken from the field-free theory. The viscosity in a magnetized matter is described by five shear viscosity coefficients. They are calculated and their dependence on the magnetic field and other parameters of dense matter is analyzed. Possible applications and open problems are outlined.

Introduction. – Shear viscosity is important in neutron stars. It regulates dissipation of hydrodynamical motions in these stars, for instance, relaxation of differential rotation to a rigid-body rotation, damping of various waves and oscillations. In particular, it can damp instabilities associated with the emission of gravitational waves (e.g., r-mode instability [1, 2]) which is important for planning advanced gravitational wave experiments.

The shear viscosity has been studied in different layers of neutron stars, in the crust and the core, but neglecting the effects of the magnetic fields. However, neutron stars can possess strong magnetic fields [3]. Typical surface fields B of ordinary neutron stars (e.g., radio pulsars) can be as high as $10^{12} - 10^{13}$ G [4], whereas the surface fields of highly magnetized neutron stars (magnetars) can be more than one order of magnitude larger [5]. The internal fields can be even stronger. To the best of our knowledge, the only calculation of the shear viscosity in a magnetic field related to compact stars was done by Haensel and Jerzak [6] for strange quark stars.

We consider the electron shear viscosity in a magnetized neutron star crust. The electrons are important momentum carriers there [7]; the basic electron scattering mechanism is the Coulomb scattering off atomic nuclei. The electrons mainly constitute a strongly degenerate relativistic and weakly interacting gas. The viscosity in the field-free case was analyzed by Flowers and Itoh [7] and detailed in [8–10]

Our analysis is based on the calculations [9] of the electron shear viscosity in the neutron star crust neglecting the magnetic field. The field makes the electron transport anisotropic which

is well studied for the cases of electron electric and thermal conductivities [11]. Here we investigate the effect of magnetic fields on the shear viscosity.

Formalism. – The electron distribution function is taken in the form

$$f(\mathbf{p}) = f_0(\epsilon) + \delta f(\mathbf{p}), \quad (1)$$

where ϵ and \mathbf{p} are, respectively, the electron energy (with the rest-mass contribution) and momentum; $f_0(\epsilon)$ is the equilibrium Fermi-Dirac distribution, and $\delta f(\mathbf{p})$ is a nonequilibrium correction.

The calculation of $\delta f(\mathbf{p})$ is based on the linearized Boltzmann equation within the relaxation time approximation of the collision integral. This equation can be written as

$$\left(\frac{\partial f_0}{\partial \mu} \right) \left(v_\alpha p_\beta \frac{\partial V_\alpha}{\partial x_\beta} - \frac{1}{3} v_\alpha p_\alpha \operatorname{div} \mathbf{V} \right) = -\frac{\delta f}{\tau} + \frac{e}{c} (\mathbf{v} \times \mathbf{B}) \frac{\partial \delta f}{\partial \mathbf{p}}, \quad (2)$$

where \mathbf{B} is the magnetic field, μ is the electron chemical potential, \mathbf{v} the electron velocity, and

$$V_{\alpha\beta} = \frac{1}{2} \left(\frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial V_\beta}{\partial x_\alpha} \right), \quad (3)$$

\mathbf{V} being the velocity of matter elements (assumed to be small, particularly, non-relativistic). Furthermore, $e = |e|$ is the elementary charge, c is light speed, and $\tau = \tau(\epsilon)$ is an effective electron relaxation time. Equation (2) is similar to that for a non-relativistic non-degenerate plasma [12] where one should replace $f_0/(k_B T) \rightarrow \partial f_0 / \partial \mu$ (k_B being the Boltzmann constant).

Since we study strongly degenerate electrons, it is sufficient to set $\epsilon = \mu$ in all functions of ϵ which vary weakly within the thermal width of the Fermi level, $|\epsilon - \mu| \lesssim k_B T$. In addition, one can set $\partial f_0 / \partial \mu \rightarrow \delta(\mu - \epsilon)$. Similar equation was written and solved for degenerate ultra-relativistic quark plasma [6]. A simple analysis show that it is valid for degenerate electrons of any degree of relativity. It is straightforward to use [6] and obtain the results required for our case.

A viscous stress tensor in the restframe of the matter is

$$\sigma_{\alpha\beta} = -\frac{2}{(2\pi\hbar)^3} \int d\mathbf{p} v_\alpha p_\beta \delta f(\mathbf{p}). \quad (4)$$

Using the results of [12] one obtains $\sigma_{\alpha\beta} = \sum_{i=0}^4 \eta_i g_{i\alpha\beta}$, where η_0, \dots, η_4 are five coefficients of shear viscosity in a magnetic field, and

$$\begin{aligned} g_{0\alpha\beta} &= (3b_\alpha b_\beta - \delta_{\alpha\beta}) \left(V_{\gamma\delta} b_\gamma b_\delta - \frac{1}{3} \text{div} \mathbf{V} \right), \\ g_{1\alpha\beta} &= 2V_{\alpha\beta} - \delta_{\alpha\beta} \text{div} \mathbf{V} - 2V_{\alpha\gamma} b_\beta b_\gamma - 2b_\alpha V_{\beta\gamma} b_\gamma + \\ &\quad + \delta_{\alpha\beta} V_{\gamma\delta} b_\gamma b_\delta + b_\alpha b_\beta \text{div} \mathbf{V} + b_\alpha b_\beta V_{\gamma\delta} b_\gamma b_\delta, \\ g_{2\alpha\beta} &= 2(V_{\alpha\gamma} b_\beta b_\gamma + b_\alpha V_{\beta\gamma} b_\gamma - 2b_\alpha b_\beta V_{\gamma\delta} b_\gamma b_\delta), \\ g_{3\alpha\beta} &= -V_{\alpha\gamma} b_\beta b_\gamma - b_{\alpha\gamma} V_{\beta\gamma} + b_{\alpha\gamma} b_\beta V_{\gamma\delta} b_\delta + b_\alpha b_{\beta\gamma} V_{\gamma\delta} b_\delta, \\ g_{4\alpha\beta} &= -2(b_{\alpha\gamma} b_\beta V_{\gamma\delta} b_\delta + b_\alpha b_{\beta\gamma} V_{\gamma\delta} b_\delta). \end{aligned} \quad (5)$$

In this case \mathbf{b} is a unit vector along \mathbf{B} , $\delta_{\alpha\beta}$ is Kronecker's delta, $b_{\alpha\beta} = \varepsilon_{\alpha\beta\gamma} b_\gamma$, and $\varepsilon_{\alpha\beta\gamma}$ is the Levi-Civita tensor.

Making use of the results of [6] for degenerate electrons of any degree of relativity we have

$$\begin{aligned} \eta_0 &= \frac{c^2 p_F^5 \tau}{15\pi^2 \hbar^3 \mu}, \quad \eta_1 = \eta_0 \frac{1}{1 + 4x_H^2}, \quad \eta_2 = \eta_0 \frac{1}{1 + x_H^2}, \\ \eta_3 &= \eta_0 \frac{2x_H}{1 + 4x_H^2}, \quad \eta_4 = \eta_0 \frac{x_H}{1 + x_H^2}. \end{aligned} \quad (6)$$

Here p_F is the electron Fermi momentum, $x_H = \omega\tau$ is a dimensionless Hall parameter and $\omega = eBc/\epsilon = eBc/\mu$ is the electron gyrofrequency. In the ultrarelativistic limit ($\mu \approx p_F c$) these equations coincide with those derived in [6] (although η_2 in [6] contains a typo, an extra factor 4 in the denominator).

Since $\sigma_{\alpha\alpha} = 0$, all the five coefficients are, indeed, shear viscosities. The behavior of $\sigma_{\alpha\beta}$ under the interchange of indices α and β and transformation $\mathbf{B} \rightarrow -\mathbf{B}$ is in line with the Onsager relations [12].

The viscous (collisional) energy dissipation rate [erg cm⁻³s⁻¹] in a shear flow is given by $T\dot{s}_{\text{coll}} = \sigma_{\alpha\beta} V_{\alpha\beta}$, where s is a specific entropy. In our case

$$\begin{aligned} T\dot{s}_{\text{coll}} &= 3\eta_0 \left(V_{\alpha\beta} b_\alpha b_\beta - \frac{1}{3} \text{div} \mathbf{V} \right)^2 \\ &+ \eta_1 \left[2V_{\alpha\beta} V_{\alpha\beta} - (\text{div} \mathbf{V})^2 - 4V_{\alpha\beta} V_{\alpha\gamma} b_\beta b_\gamma \right. \\ &+ \left. 2V_{\alpha\beta} b_\alpha b_\beta \text{div} \mathbf{V} + (V_{\alpha\beta} b_\alpha b_\beta)^2 \right] \\ &+ 4\eta_2 \left[V_{\alpha\beta} V_{\alpha\gamma} b_\beta b_\gamma - (V_{\alpha\beta} b_\alpha b_\beta)^2 \right]. \end{aligned} \quad (7)$$

The nature of the viscosities $\eta_0 - \eta_4$ is related to the orthogonality of the tensors g_0, \dots, g_4 to the 'magnetic' tensor $b_\alpha b_\beta$.

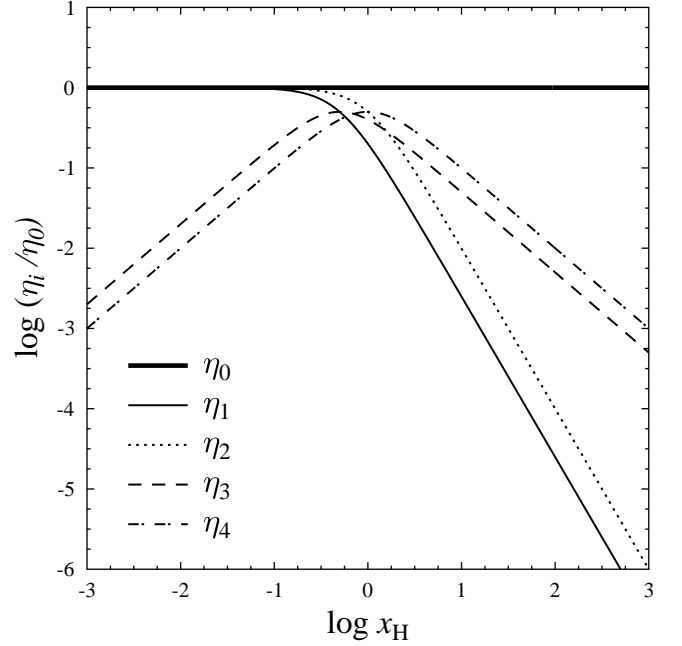


Fig. 1: Electron shear viscosity coefficients η_i ($i = 0, \dots, 4$) in units of the field-free viscosity η_0 versus the Hall parameter $x_H = \omega\tau$.

One has $g_{i\alpha\beta} b_\beta = b_\alpha g_{i\alpha\beta} = 0$ for $i=1$ and 3 , and $g_{i\alpha\beta} b_\alpha b_\beta = 0$ for $i=2$ and 4 , but

$$g_{0\alpha\beta} b_\alpha b_\beta = 2V_{\alpha\beta} b_\alpha b_\beta - \frac{2}{3} \text{div} \mathbf{V} \neq 0. \quad (8)$$

Accordingly, g_0 is 'parallel' to $b_\alpha b_\beta$, g_2 and g_4 are 'transverse' to $b_\alpha b_\beta$, whereas g_1 and g_3 are 'fully transverse' to $b_\alpha b_\beta$. Since g_3 and g_4 include $b_{\alpha\beta}$, they are pseudotensors and describe Hall-like momentum transfer.

Therefore, in analogy with the well-known classification of electric and thermal conductivities in a magnetized plasma, we can call η_0 the *longitudinal viscosity* (with respect to \mathbf{B}), η_1 the *fully transverse viscosity*, η_2 the *ordinary transverse viscosity*, η_3 the *fully Hall viscosity* and η_4 the *ordinary Hall viscosity*.

One has $\eta_1(x_H) = \eta_2(2x_H)$ and $\eta_3(x_H) = \eta_4(2x_H)$. These relations become clear after writing the system of equations for η_0, \dots, η_4 in a local reference frame with the z axis along \mathbf{B} . Such a system splits into three subsystems. The first subsystem is for η_0 which appears to be determined by V_{zz} . The 'driving force' V_{zz} is directed along \mathbf{B} and does not interfere with the Lorentz force acting on electrons. Accordingly, η_0 is formally independent of B . The second subsystem is for η_2 and η_4 with the 'driving force' created by V_{xz} and V_{yz} (partly along \mathbf{B} but partly across \mathbf{B} , interferes with ordinary electron gyrorotation with frequency ω). This subsystem gives us the ordinary transverse and Hall viscosities. Finally, the third subsystem is for η_1 and η_3 with the 'driving force' created by V_{xx} , V_{xy} and V_{yy} . Such a force is fully transverse to \mathbf{B} and interferes with the second harmonic of gyrorotation (with frequency 2ω). It gives the fully transverse and Hall viscosities. Notice, that the viscosities η_0, η_1 and η_2 are non-negative. They determine the

viscous dissipation (7) of motions of the matter. In contrast, the Hall viscosities η_3 and η_4 may have different sign (depending on the sign of electric charge of momentum carriers) and do not contribute to the viscous dissipation (7).

Fig. 2 shows the dependence (6) of all the five viscosities on the Hall parameter $x_H \propto B$. At weak fields B , where $x_H \ll 1$, the electrons are non-magnetized (their collision frequency $1/\tau$ is much higher than ω). In this case the transverse viscosities η_1 and η_2 converge to η_0 , and the Hall viscosities behave as $\eta_3 \approx 2x_H\eta_0$ and $\eta_4 \approx x_H\eta_0$. The viscous stress tensor $\sigma_{\alpha\beta}$ takes the form which is well-known in a non-magnetized plasma [12]. In the case of moderately magnetized electrons, $x_H \sim 1$, all five viscosities are of the same order of magnitude. At higher B , when $x_H \gg 1$, the electrons are strongly magnetized. Then $\eta_1 \approx \eta_0/(2x_H)^2$, $\eta_2 \approx \eta_0/x_H^2$, $\eta_3 \approx \eta_0/(2x_H)$ and $\eta_4 \approx \eta_0/x_H$. In this regime the electrons frequently rotate about \mathbf{B} -lines and rarely scatter off atomic nuclei.

Viscosity in a neutron star crust. – The parameters of electrons and atomic nuclei in a neutron star crust are provided by the models of the crust, e.g. [13]. For illustration, we take standard models with one type of nuclei at any mass density ρ . We will mainly use the model of the ground-state (cold catalyzed) crust for the BSk21 equation of state approximated analytically in [14].

We employ the effective electron relaxation time [9] which determines the field-free shear viscosity η_0 . It seems to include the most elaborated physics input. It is valid for gaseous, liquid and crystalline states of atomic nuclei. It takes into account proper plasma screening of the electron-nucleus interaction, multiphonon processes in the crystalline phase in the harmonic lattice approximation, and finite sizes of atomic nuclei. However, it neglects electron band structure effects which can be important at sufficiently low temperatures. If $B = 0$, our consideration is valid at the same conditions as in [9] but at strong fields the validity conditions are more restrictive (see below).

The electron relaxation time can be written as (e.g. [9]),

$$\tau = \frac{p_F^2 v_F}{12\pi Z^2 e^4 n_i \Lambda}, \quad (9)$$

where n_i is the number density of atomic nuclei and Λ is an effective Coulomb logarithm for electron-nucleus scattering. It was analytically approximated in [9] using the method of an effective electron-nucleus potential first implemented [15] for the electron conduction problem. The approximation is valid for a broad class of spherical atomic nuclei which can be available in the neutron star crust.

Fig. 2 shows the density dependence of the electron Hall parameter x_H through the neutron star crust with the BSk21 equation of state. Small jumps of the nuclear composition due to changes of nuclides with increasing density ρ are smoothed out as described in [13, 14]. The only jump which is left occurs in the neutron-drip point ($\rho_{ND} \approx 4.3 \times 10^{11} \text{ g cm}^{-3}$) shown by the vertical dotted line. It divides the neutron star crust into the outer and inner crust. The inner crust disappears (transforms

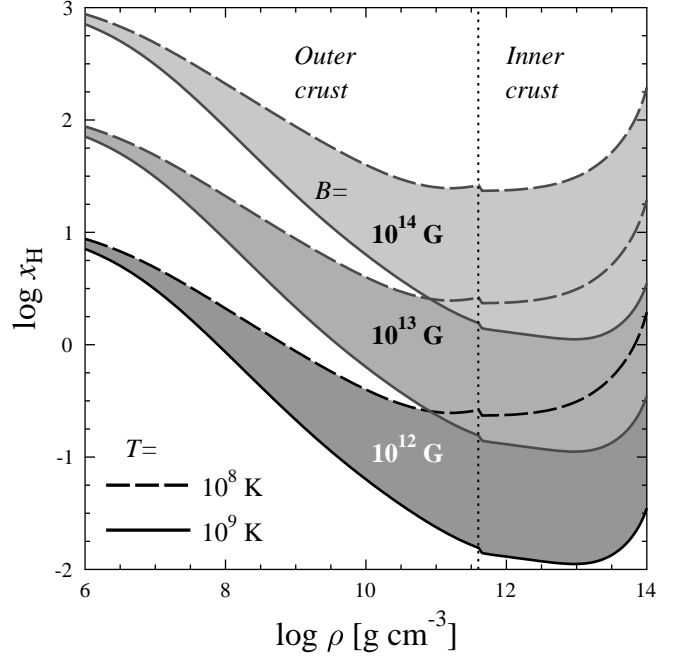


Fig. 2: Hall parameter x_H versus mass density ρ in a neutron star crust for three values of the magnetic field, $B = 10^{12}, 10^{13}$ and 10^{14} G, and two temperatures, $T = 10^8$ and 10^9 K. When B is fixed and the temperature decreases from 10^9 to 10^8 K, an $x_H(\rho)$ curve increases from an appropriate solid to a dashed line. The vertical dotted line shows the neutron drip point. See text for details.

into the liquid core) at $\rho \approx 1.4 \times 10^{14} \text{ g cm}^{-3}$, just after the highest density displayed in Fig. 2.

The displayed Hall parameter is a proper measure of magnetic effects on the electron shear viscosity. It is shown for three values of $B = 10^{12}, 10^{13}$, and 10^{14} G. Magnetic fields $B \sim 10^{12} - 10^{13}$ G are typical for ordinary pulsars, whereas higher B are more typical for magnetars. We have taken two temperatures, $T = 10^9$ (solid lines) and 10^8 K (dashed lines), characteristic for ordinary young isolated (cooling) neutron stars and magnetars. If B is fixed and the temperature in the star falls down from 10^9 to 10^8 K, the relaxation time grows and an $x_H(\rho)$ curve goes up from a solid to a dashed line amplifying the electron magnetization. At the lowest displayed density $\rho = 10^6 \text{ g cm}^{-3}$ the relaxation time is almost temperature independent, so that the solid and dashed curves converge. The decrease of the electron magnetization with increasing ρ at fixed B in the outer crust is mainly provided by the decrease of the electron gyrofrequency ω due to growing μ . One sees that at $B = 10^{12}$ G the electrons stay weakly magnetized (in the given T range) throughout the entire crust except for the surface layers. At $B = 10^{13}$ G, the electrons become strongly magnetized but only at $\rho \lesssim 10^{10} \text{ g cm}^{-3}$, whereas in the inner crust they are moderately magnetized. If $B = 10^{14}$ G, the electrons in the outer crust are mostly strongly magnetized, while in the inner crust they are strongly magnetized only at $T \lesssim 10^8$ K. As expected, the strongest magnetization takes place at the lowest densities.

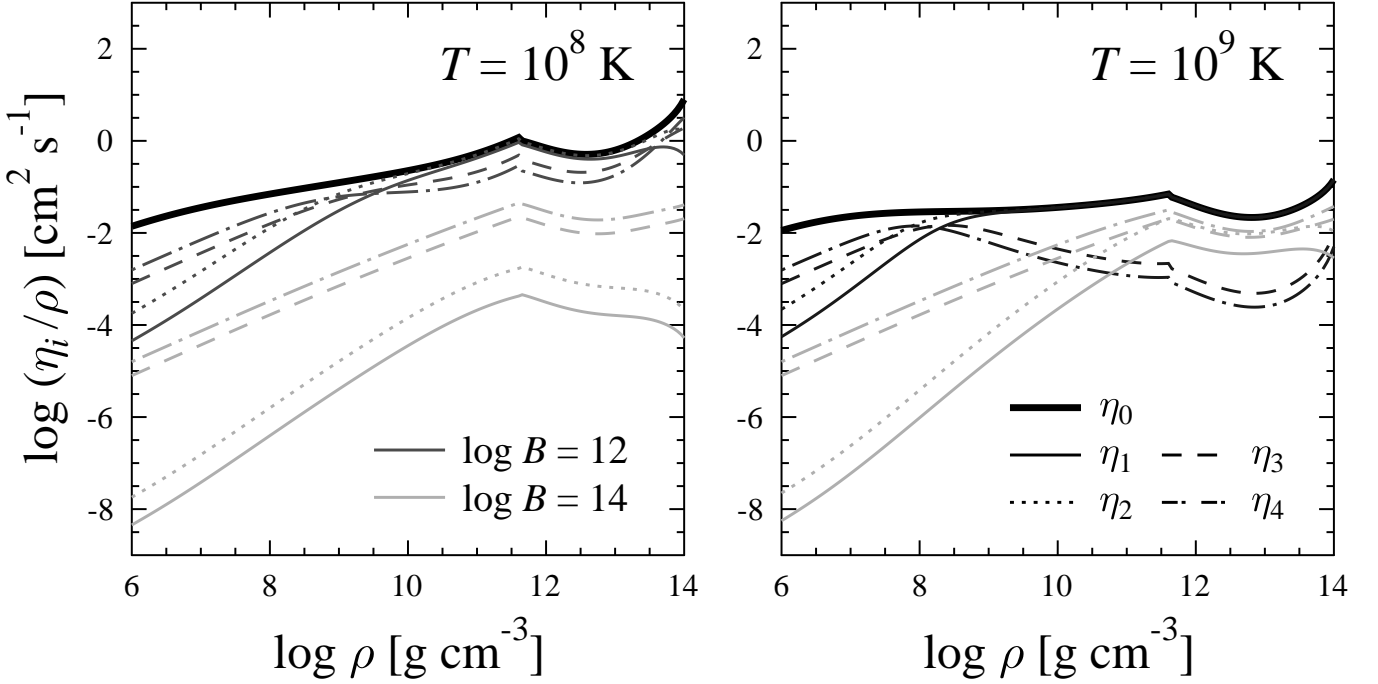


Fig. 3: Density dependence of kinematic electron shear viscosities η_i/ρ ($i = 0, \dots, 4$) for two values of $B = 10^{12}$ and 10^{14} G at two temperatures $T = 10^8$ K (left) and 10^9 K (right). See text for details.

Fig. 3 shows the density dependence through the entire crust of all five electron viscosities η_0, \dots, η_4 (lines of different type and darkness). Calculations are done in the same formalism as in Fig. 2. Because the main trend is that $\eta_0 \propto \rho$, we show the kinematic viscosities η_i/ρ . The left panel of Fig. 3 corresponds to $T = 10^8$ K, while the right panel is for $T = 10^9$ K. The thick black line in each panel gives the field-free viscosity η_0/ρ . Moderately dark thinner lines present the viscosities η_1, \dots, η_4 for $B = 10^{12}$ G, whereas light lines display these viscosities for $B = 10^{14}$ G. The behavior of the viscosities can be easily understood from Figs. 1 and 2. At $B = 10^{14}$ G the electron magnetization is generally strong and the transverse viscosity components are the lowest one. If $B = 10^{12}$ G, the magnetization is weaker and the Hall viscosities could be the lowest ones.

So far we have considered only the crust made of the BSk21 equation of state. We have checked also the BSk20 and BSk19 equations of state, taking analytical approximations from [14], and the smooth composition model [13] of the crust. The results appear to be almost the same.

As mentioned above, our consideration of the transverse and Hall viscosities η_1, \dots, η_4 is limited by the relaxation time approximation of the collision integral in the Boltzmann equation. This approximation is valid [16] as long as typical electron energy transfers in electron-nucleus collisions are smaller than the thermal energy width $\sim k_B T$ of the electron Fermi level. If so, the Wiedemann-Franz relation between the electric and thermal conductivities of electrons is fulfilled, which can be used as the validity condition. Therefore, we have used our conductivity code [15, 17] and checked the Wiedemann-Franz

relation numerically. If $T \gtrsim 10^9$ K, this relation is reasonably well fulfilled throughout the entire crust meaning that our consideration is justified. At $T \sim 3 \times 10^8$ K, it is well fulfilled for $\rho \lesssim 10^{13} \text{ g cm}^{-3}$ but violated at higher ρ within a factor of about 3. At $T \sim 10^8$ K, it works reasonably well for $\rho \lesssim 10^{12} \text{ g cm}^{-3}$, but violated at higher ρ within a factor of 10. Therefore, our consideration is strictly justified at not too high densities and not too low temperatures; otherwise it can be treated as semi-quantitative. It is possible to improve the theory by using proper treatment of inelastic electron collisions but it is beyond the scope of this work.

We have also neglected some other effects which might affect the results. For instance, at temperatures $T \lesssim T_B = \hbar \omega_B / k_B$ (ω_B being the cyclotron frequency of the nuclei) the frequencies of phonons responsible for electron transport in Coulomb crystals of atomic nuclei will be affected by magnetic fields and change the relaxation time τ . As seen, for instance, from Fig. 3 in [18], T_B is much lower than T for the conditions considered above and the effect is not important here.

Note that at $T \lesssim 3 \times 10^7$ K, along with the electron-nucleus scattering, an additional mechanism of electron scattering by charged impurities may become important. It has been widely discussed for electron electric and thermal conductivities, e.g. [15], and it was also discussed for the field free shear viscosity [9]. It is easily incorporated into the present formalism by using the combined relaxation time τ given by $\tau^{-1} = \tau_{\text{ei}}^{-1} + \tau_{\text{e-imp}}^{-1}$, where τ_{ei} refers to electron-nucleus collisions considered above, and $\tau_{\text{e-imp}}$ refers to electron scattering by impurities [9]. We do not discuss this case here because of the lack of space and because the number density and electric charges

of the impurities are largely unknown. At very low temperatures the relaxation is fully dominated by electron-impurity collisions, $\tau = \tau_{e\text{-imp}}$. These collisions are thought to be elastic [7, 9] and described in the relaxation time approximation. At intermediate temperatures, both mechanisms, the electron-nucleus and electron-impurity collisions, are equally important and the problem is complicated by the description of inelastic electron-nucleus collisions.

In addition, electron-electron collisions can also contribute to the shear viscosity. In the field-free case this contribution was studied in [10] and found to be rather unimportant. To the best of our knowledge, the shear viscosity due to electron-electron collisions in a magnetized plasma of neutron stars has not been studied in the literature.

White dwarfs. – The above results are equally valid for degenerate cores of white dwarfs. The density range there is lower ($\rho \lesssim 10^{10} \text{ g cm}^{-3}$) and the nuclear composition is more restrictive (mainly ^4He , ^{12}C , ^{16}O). In such a plasma the relaxation time approximation for describing the electron nucleus collisions is valid to lower temperatures, down to a few million K for a typical density $\rho \sim 10^6 \text{ g cm}^{-3}$. At this density, in the field $B = 10^9 \text{ G}$ the electrons are weakly magnetized, so that $\eta_0 \approx \eta_1 \approx \eta_2$, whereas the Hall viscosities η_3 and η_4 are lower than η_0 by about one order of magnitude. If $B = 10^{10} \text{ G}$, the electrons become moderately magnetized, and at $B = 10^{11} \text{ G}$ their magnetization becomes strong, with η_1 lower than η_0 by more than two orders of magnitude. These effects can affect viscous damping of oscillations of magnetic white dwarfs [19, 20].

Discussion and conclusions. – We have calculated the electron shear viscosity due to collisions of electrons with atomic nuclei in a magnetized neutron star crust. Calculations are done for strongly degenerate relativistic or non-relativistic electrons. The collision integral in the Boltzmann equation is taken in the relaxation time approximation, but the effective relaxation time τ is taken from more advanced field-free calculations [9]. The shear viscosity in a magnetic field B is described by the five viscosity coefficients, $\eta_0 - \eta_4$. In our approximation, η_0 is independent of B and plays the role of the viscosity along B . Other viscosities, transverse to B (η_1 and η_2) and Hall ones (η_3 and η_4) do depend on B . All these viscosities are presented in the form convenient for computing using any realistic composition of the neutron star crust. The viscosity η_0 is valid for the same conditions as in [9]; other viscosities are restricted by the applicability of the relaxation time approximation.

The dependence of the shear viscosities on B is determined by the Hall parameter x_H . Generally, the transverse and Hall viscosities are lower than η_0 . At high electron magnetization, $x_H \gg 1$, they become much lower than η_0 . The magnetization increases with lowering ρ and T (Fig. 2). It is much easier to magnetize the electrons in the outer neutron star crust than in the inner one. At $T \sim 10^9 \text{ K}$ the electrons in the inner crust remain weakly or moderately magnetized even by magnetars' fields $B \sim 10^{14} \text{ G}$.

The shear viscosity is important for modeling many phenom-

ena in neutron stars, particularly, the relaxation of differential rotation to a rigid-body rotation or the damping of waves and oscillations of the stars, including the damping of instabilities accompanied by the emission of gravitational waves. Since the transverse and Hall viscosities are lower than η_0 , one can expect that the viscous damping of oscillating motions in a magnetized neutron star crust will not significantly exceed the viscous damping of similar motions in a non-magnetic crust [9]. The anisotropic shear viscosity in magnetized matter may produce noticeably different damping times for motions of different geometry and orientation with respect to the magnetic field. The present results are equally valid for magnetic white dwarfs.

We have analyzed the basic features of the electron shear viscosity in a neutron star crust. However, there remain a number of problems to be solved. First, it would be important to extend the calculations to sufficiently high densities and low temperatures where the electron-nucleus scattering becomes inelastic and the relaxation time approximation breaks down. It would be good to include into these calculations the electron band structure effects in crystalline matter (in analogy to the electron conduction problem [21]) and consider mixtures of different nuclei. All this is feasible but requires a lot of effort, a good project for future work. Second, it would be important to study the case of very strong magnetic fields ($T \lesssim T_B$, see above) which affect the vibration properties of atomic nuclei in dense matter and introduce the dependence of the relaxation time on B . One may expect, that in this case the viscosity coefficients will contain three different relaxation times (one for η_0 , the second for η_1 and η_3 , and the third one for η_2 and η_4). If the bottom of the crust contains the layer of exotic nuclear clusters (the so called nuclear pasta) the electron shear viscosity there will be more complicated (in analogy with a more complicated conductivity [22]). Finally, very strong magnetic fields can quantize electron states into Landau orbitals, and the viscosity coefficients may show quantum oscillations due to the population of new Landau levels with increasing density (similarly to quantum oscillations of electron conductivities [23]).

In addition, one should take into account other sources of shear viscosity in neutron star envelopes. In particular, the main contribution in the surface layers of neutron stars (roughly at $\rho \lesssim 10^4 \text{ g cm}^{-3}$) comes from ions (atomic nuclei). This viscosity for a non-degenerate weakly coupled plasma of electrons and ions in a magnetic field was calculated in [24] with similar results on the existence of five shear viscosities of ions. However, physical conditions in the outer layers of neutron stars are more complicated because the ions there can be strongly coupled and partially ionized. The effect of strong coupling on the shear viscosity of ions at $B = 0$ has been studied (e.g. [25, 26]) but the entire problem has not been solved. In the inner crust of a neutron star some contribution to the shear viscosity may come from free neutrons which interact with the atomic nuclei and can be in superfluid state. Moreover, in addition to the shear viscosity, there is the bulk viscosity of dense matter which can be associated with weak interactions and have entirely different physical properties (in analogy with the bulk viscosity in neutron star cores [27]). All these problems are almost not considered and constitute an open field of physical kinetic of

neutron stars.

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